Dispersion of solid heavy particles in a homogeneous turbulence

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Abstract

Prediction of solid heavy particle dispersion in turbulent flows requires the knowledge of the fluid velocity seen by the solid particle. A one particle and one time scale lagrangian stochastic model has been developed for the determination of the driving fluid velocity. A modified time correlation taking into account the effects of gravity and inertia is used. We focus on the grain trajectories of saltating particles whose volumetric mass are much greater than that of the fluid $(\rho_p / \rho_f \geq 10^3)$. A comparison with wind tunnel experiments run by Snyder and Lumley [5] is made. The good agreement of the numerical results with experimental data demonstrates that the eolian transport must be considered as a stochastic process. This approach is promising and quite well adapted for the modelisation of eolian saltating particles.

1 Introduction

Dust produced by wind erosion is a major source of atmospheric aerosols. On the basis of wind-tunnel observations [1], it has been found that saltation bombardment appears to be mainly responsible to dust emission. During an erosion event, saltation can move large quantities of soil over distances from meters to kilometers. In the process, the soil is winnowed of its finer particles. These small particles become suspended in the air as dust, and can be dispersed away from the surface by atmospheric turbulence and finally transported over very large distances. So, the study of the dispersion of

small saltating solid heavy particles in turbulent flows is important for understanding this natural phenomena.

If we consider the dispersion of heavy solid particles in air, the flow will interact with the particles through different forces [2]. If the particle-fluid density ratio is much greater than one $\left(\rho_p/\rho_a \geq 10^3 \right)$ which is the case for sand particles for example $(\rho_{sand} \approx 2600\,kg/m^3)$, we may neglect all forces except the gravity and the viscous forces. The simplified equations for a nonrotating particle reduce to

$$\begin{cases} \dfrac{dV_i(t)}{dt} = \dfrac{\left[u_i^p(\vec{y}(t),t) - V_i(t) \right]}{\tau_p} f - g\delta_{i1} \\ \dfrac{dy_i(t)}{dt} = V_i(t) \end{cases} \tag{1}$$

where $V_i(t)$ and $y_i(t)$ refer respectively the particle velocity and location, g is the gravitational acceleration aligned with the "1" direction and $\tau_p = \dfrac{\rho_p d_p^2}{18\mu}$ represents the relaxation time of the particle. f is a coefficient which can be expressed following Clift et al. [3] as $f = 1 + 0,15\Re e_p^{0,687}$.

$\Re e_p$ refers the particle Reynolds number, $\Re e_p = \dfrac{u_{rel}d_p}{\nu}$, u_{rel} and d_p refer to the particle relative velocity and diameter respectively and ν denotes the fluid kinematic viscosity.

The motion of particles in a statistically homogeneous isotropic turbulence has been studied in order to clarify the effects of the particle inertia and the particle drift velocity due to the gravitational force on the dispersion (Csanady [4], Snyder and Lumley [5], Wells and Stock [6], Squirre and Eaton [7]). The main difficulty in resolving the equations lies in the determination of the fluid velocity $u_i^p\left[\vec{y}(t),t\right]$. This driving velocity corresponds to the velocity of the fluid element following the solid particle trajectory or the fluid velocity seen by the solid particle. Due to the gravity and the inertia effects, the fluid and the solid particles will be decorrelated, these particle and fluid velocities will be different and a relative velocity will appear. One can also expect that the solid particle will move out the initial turbulent eddy and enter a new one.

In that study, we estimate the driving fluid velocity using the stochastic approach. Eolian sediment transport must be considered a stochastic process because two main reasons. First of all, the particles are transported by turbulent fluctuations of the wind and second, the impacts of saltating particles with soil results in a subsequent ejections whose number and velocities may be known only in a probabilistic sense.

2 Modelisation of the driving fluid velocity

We consider for simplicity the one dimensional case. For a statistically homogeneous and isotropic turbulence, the one dimensional stochastic equation writes (Thomson [8])

$$du_2(t) = -\frac{u_2(t)}{T_L}dt + \sigma_{u_2}\sqrt{\frac{2}{T_L}}d\zeta(t) \tag{2}$$

where $u_2(t)$ and σ_{u_2} denote the fluid velocity and rms velocity respectively, T_L the fluid lagrangian integral time scale and $d\zeta(t)$ a Wiener increment. We have to evaluate an integral correlation time for the driving fluid velocity along the solid particle trajectory. This time scale is different from the fluid lagrangian time scale and we may expect it to be smaller. A general form could be expressed as

$$T_L^p = \frac{T_L}{\alpha_{grav} + \alpha_{inert}}$$

where the two coefficients α_{grav} and α_{inert} represent the gravity and inertia effects coefficients respectively. The gravity effect is estimated following the parametrisation proposed by Csanady [5] when the inertia is negligible, i.e.

$$T_p = \frac{T_L}{\left[1+\left(\frac{\alpha V_{\lim}T_L}{L}\right)^2\right]^{1/2}} = \frac{T_L}{\alpha_{grav}}$$

with $\alpha = 1$ if u_2 parallel to g and $\alpha = 2$ if u_2 perpendicular to g. Second, considering inertia as a characteristic for particles to repond to the turbulence frequencies, the inertia coefficient α_{inert} may be expressed as the ratio of the relative particle-fluid velocity to the rms fluid velocity, i.e.

$$\alpha_{inert} = \frac{|u_{2rel}|}{\sigma_{u_2}} = \frac{|u'_2 - V_2|}{\sigma_{u_2}}$$

The modified correlation time T_L^p will then writes

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$$T_L^P \doteq \cfrac{T_L}{\left[1+\left(\cfrac{\alpha V_{\lim} T_L}{L}\right)^2\right]^{1/2} + \cfrac{|u'_2 - V_2|}{\sigma_{u_2}}} \tag{3}$$

It is this new correlation time T_L^P which modifies the stochastic equation to represent the fluid velocity along a particle trajectory. In a discretized form, the stochastic equation becomes

$$u_{2n+1}^P = \left(1 - \frac{\Delta t}{T_L^p}\right) u_{2n}^P + \sigma_{u_{2n-1}} \sqrt{1 - \left(1 - \frac{\Delta t}{T_L^P}\right)^2} \; \chi_{n+1}^P$$

$$u_{2n+1}^P = \left(1 - \frac{\Delta t}{T_p} - \frac{|\Delta r_2|}{L}\right) u_{2n}^P + \sigma_{u_{2n+1}} \sqrt{1 - \left(1 - \frac{\Delta t}{T_p} - \frac{|\Delta r_2|}{L}\right)^2} \; \chi_{n+1}^P \tag{4}$$

where Δt denotes the time step. Δr_2 is the separation between the driving fluid velocity element at the present time t_{n+1} and at the previous time t_n, u_{2n} is the fluid velocity at time t_n and L is the spatial integral scale. Our model differs from that of Zhuang et al. [9] in the use of T_p in place of T_L. In our model, the gravity effect is better estimated.

In consistency with the one particle-one time scale stochastic equation, the correlation coefficient writes under an exponential form as

$$R_{22}^P(\Delta t) = \exp\left\{-\frac{\Delta t}{T_L^P}\right\} = \exp\left\{-\frac{\Delta t}{T_L}\left(\sqrt{1 + \left(\frac{\alpha V_{\lim} T_L}{L}\right)^2} + \frac{|u'_2 - V_2|}{\sigma_{u_2}}\right)\right\}$$

$$R_{22}^P(\Delta t) = \exp\left\{-\frac{\Delta t}{T_p} - \frac{|\Delta r_2|}{L}\right\}$$

3 Numerical results

Snyder and Lumley [5] measured the dispersion of several types of particles injected at a point $x_{1,0} = 20M$ downstream the grid into a decaying homogeneous isotropic turbulence in a wind tunnel. The mean stream direction (x_1) was vertical, aligned with the gravitational acceleration g and

the grid Reynolds number about 10^4. The measurements of dispersion perpendicular to the mean flow represent dispersion (x_2) lateral to the direction of the gravitational force. At a downstream location of $x_1 / M = 73$, the integral time scale is $T_L = 0.141 s$ and the Kolmogorov microscales are $\tau_k = 11.6 ms$ and $\eta_k = 0.043 cm$ The particle characteristics are summarized on the table 1.

Table 1. Characteristics of the particles used by Snyder and Lumley [5]

	Hollow glass	Glass	Corn	Copper
Diameter d_p (µm)	45.6	87.0	87.0	45.6
Density ((kg/m³)	260	2500	1000	8900
Characteristic time scale measured (ms) τ_p	1.7	45.0	20.0	49.0
d_p / η_k	0.105	0.198	0.198	0.105
Particle Reynolds number	0.05	2.48	1.10	1.45
Drift velocity $V_g = \tau_p g$	0.0167	0.441	0.196	0.481
Time ratio τ_p / τ_k	0.145	3.85	1.72	4.21
Time ratio τ_p / T_L	0.012	0.32	0.142	0.347
V_g / σ_{u_2}	0.127	3.37	1.5	3.67

The particle eqns (1) were solved using a second-order Runge-Kutta scheme, the fluid driving velocity was determined using the discretized stochastic equation system

$$\begin{cases} u_{1\,n+1} = U = cte \\ u_{2\,n+1}^P = \left(1 - \dfrac{\Delta t}{T_L^P}\right) u_{2\,n}^P + \sigma_{u_{2n+1}} \sqrt{1 - \left(1 - \dfrac{\Delta t}{T_L^P}\right)^2} \chi_{n+1}^P \end{cases}$$

The separation Δr_2 between the present and the previous driving fluid particles is determined using a stochastic homogeneous velocity eqn (2).

Figures 1 and 2 show a comparison of our model calculations with the laboratory data of Snyder and Lumley [5]. They presented results for the dispersion of particles relative to their displacement at station $x_1 = 48.4 M$

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downstream of the source. They measured the dispersion $[y(t) - y(t_1)]^2$ where t_1 is the travel time to station 1, t the travel time to subsequent downstream stations and y(t) is the lateral displacement of a particle at time t. This is equivalent to the dispersion from a source at station 1. They also measured the lateral particle velocity variance as a function of distance downstream the source. For our calculations, 40 000 particles were released. We note a very good agreement for the dispersion and an acceptable agreement for the velocity variance, specially for the 3 heavier particles. According to Taniere et al. [10], saltation behavior can be defined when the quantities τ_p / T_L and V_g / σ_{u_2} are greater than one. The particles of corn, glass and copper clearly exhibit a saltation or a modified saltation behavior (see table 1).
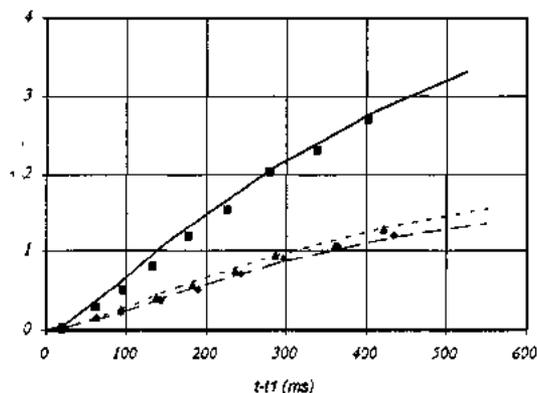

Figure 1:   Comparison of present calculations (lines) with Snyder and Lumley [5] data (symbols) for the particle dispersion $[y(t) - y(t_1)]^2$. Corn, solid lines and squares ; Copper, large dashed lines and circles ; glass, dashed lines and triangles.
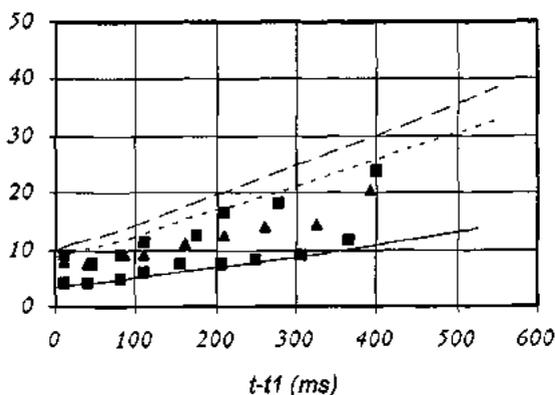
Figure 2:   Comparison of present calculations (lines) with Snyder and Lumley [5] data (symbols) for the lateral particle velocity variance $\overline{U}^2 / \sigma_{V_2}$. Corn, solid lines and squares ; Copper, large dashed lines and circles ; glass, dashed lines and triangles.
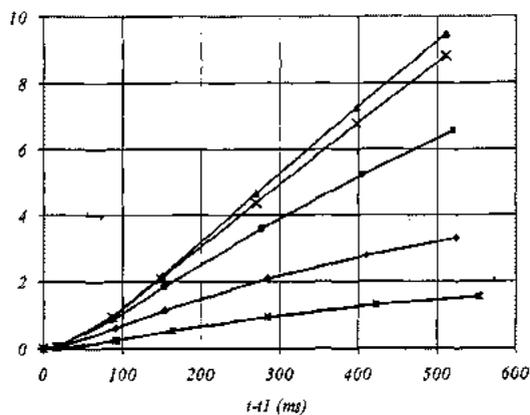

Figure 3:   Comparison of present calculations (lines) for dispersion $[y(t) - y(t_1)]^2$ of various particles. Profiles from bottom to top : solid glass, $d_p = 87.4\,\mu m$; corn, $d_p = 87.4\,\mu m$; corn, $d_p = 46.5\,\mu m$; hollow glass, $d_p = 46.5\,\mu m$; passive scalar.

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On the other hand, the lightest hollow glass particles which verify $\tau_p / \tau_k < 1$ exhibit a pure suspension behavior. These particles are out the scoop of our present study. With such small gravity and inertia parameters, the calculations for hollow glass particles are very close to the passive scalar dispersion. Figure 3 depicts the dispersion of various particles with different densities and diameters. For light particles our model correctly reverts to passive scalar.

Figures 4 and 5 present the mean concentration profiles in different locations downstream the source for the corn and the glass particles respectively. The shape of all the concentration profiles is Gaussian. The effects of density are clearly enhanced, the greater is the density, the lower is the dispersion. More, it seems that the mean concentration <C> is self similar (figure 6). Here, the mean concentration is nondimensionalised by the maximum of concentration on the axis and the ordinate y by the width σ_m of the gaussian which better fit the curves, i.e.

$$\frac{<c>}{<c>_{max}} = \exp\left\{-\frac{1}{2}\left(\frac{z}{\sigma_m}\right)^2\right\}.$$
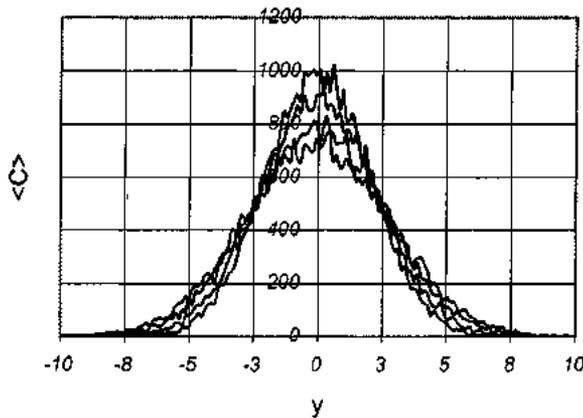

Figure 4: Mean concentration profiles of the corn particles in four locations downstream the source, $x_1 / M = 68.4, 90.5, 138, 200$.
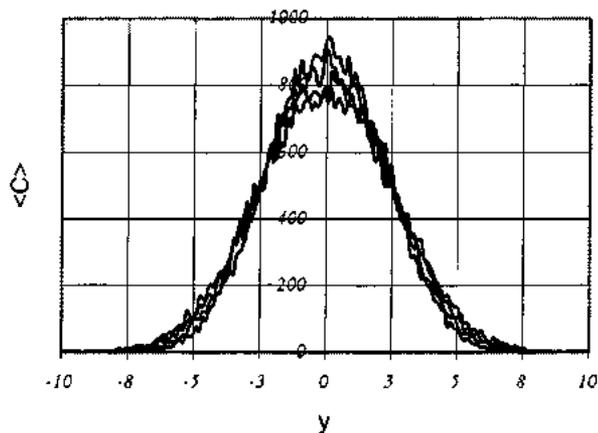
Figure 5: Mean concentration profiles of the glass particles in four locations downstream the source, $x_1 / M = 68.4, 90.5, 138, 200$.
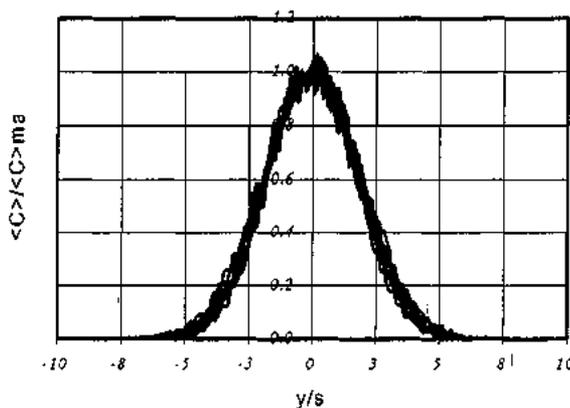

Figure 6: Nondimensionalised mean concentration profiles of the corn, glass and copper particles in the four locations $x_1 / M = 68.4, 90.5, 138, 200$, downstream the source.

4 Conclusion

We have attempted to address the effects of gravity and inertia on the fluid driving velocity involved in solid particle transportation. A lagrangian stochastic model has been developed using a new time scale correlation. The model calculation fit quite well with the experimental results for saltating

particles in homogeneous flows. This promising work presented here will be extended to solid particle transportation in atmospheric boundary layers.

References

[1] Shao, Y. & Raupach, M.R., *Effect of saltation bombardment on the entrainment of dust by wind*, J. of Geophysical Research, 98, D7, pp 12719-726, 1993.

[2] Maxey, M.R. & Riley, J.J., *The equation of motion for small rigid particle sphere in a non uniform flow*, Phys Fluids, 26, 883-889, 1983.

[3] Clift, R., Grace, J.R. & Weber M.E., *Bubbles, drops and particles*, New York Academic Press, 1978.

[4] Csanady, G.T., *Turbulent diffusion of heavy particles in the atmosphere*, J. Atmos. Science, 20, pp 201-208, 1963.

[5] Snyder, W.H. & Lumley, J.L., *Some measurements of particle velocity autocorrelation functions in a turbulent flow*, J. Fluid Mech., vol. 48, pp 41-71, 1971.

[6] Wells, M.R. & Stock, D.E., *The effects of crossing trajectories on the dispersion of particles in a turbulent flow*, J. Fluid Mech., vol. 136, pp 2758-2776, 1983.

[7] Squirre, K.D. & Eaton, J.K., Measurements of particle dispersion obtained from direct numerical simulations of isotropic turbulence, J. Fluid Mech., vol. 226, pp 1-35 1991.

[8] Thomson, D.J., *Criteria for the selection of stochastic models of particle trajectories in turbulent flows*, J. Fluid Mech., vol.180, pp 529-556, 1987.

[9] Zhuang, Y., Wilson, J.D. & Lozowski, E.P., *A trajectory simulation model for heavy particle motion in turbulent flow*, J. of Fluid Engineering, vol 111, pp 492-494, 1989.

[10] Taniere, A., Oesterle, B.& Monnier, J.C., *On the behavior of solid particles in a horizontal boundary layer with turbulence and saltation effects*, Exp. In Fluids, 23, pp 463-471, 1997.